

Chapter 1

Finite Elements Overview

1.1 Modeling Basics

“There are no exact answers. Just bad ones, good ones and better ones. Engineering is the art of approximation.” Approximation is performed with models. We **consider** a reality of interest, e.g., a concrete beam. In a first view, it has *properties* such as dimensions, color, surface texture. From a view of structural analysis the latter ones are irrelevant. A more detailed inspection reveals a lot of more properties: composition, weight, strength, stiffness, temperatures, conductivities, capacities, and so on. From a structural point of view some of them are essential. We combine those essential properties to form a *conceptual model*. Whether a property is essential is obvious for some, but the valuation of others might be doubtful. We have to choose. By choosing properties our model becomes approximate compared to reality. **Approximations** are more or less accurate.

On one hand, we should reduce the number of properties of a model. Any reduction of properties will make a model less accurate. **Nevertheless, it might remain a good model.** On the other hand, an over-reduction of properties will make a model inaccurate and therefore useless. Maybe also properties are introduced which have no counterparts in the reality of interest. Conceptual modeling is the art of choosing properties. As all other arts it cannot be performed guided by strict rules.

The chosen properties have to be related to each other in quantitative manner. This leads to a *mathematical model*. In many cases, we have systems of differential equations relating variable properties or simply *variables*. After prescribing appropriate boundary and initial conditions an exact, unique solution should exist for variables depending on spatial coordinates and time. Thus, a particular variable forms a field. Such fields of variables are infinite as space and time are infinite.

As analytical solutions are not available in many cases, a discretization is performed to obtain approximate numerical solutions. *Discretization* reduces underlying infinite space and time into a finite number of supporting points in space and time and maps differential equations into algebraic equations relating a finite number of variables. This leads to a *numerical model*.